Reviews on Advanced Materials and Technologies, 2023, vol. 5, no. 3, pp. 30–38
DOI: 10.17586/2687-0568-2023-5-3-30-38

Constitutive Model of Rock, Nonlinearity and Localization

Yu.P. Stefanov[1,2]

[1] Trofimuk Institute of Petroleum Geology and Geophysics SB RAS, Koptug ave. 3, 630090, Novosibirsk, Russia
[2] Institute of Strength Physics and Materials Science SB RAS, pr. Akademicheskii, 2/4, 634055, Tomsk, Russia

Received: August 28, 2023 Corresponding author: Yu.P. Stefanov

Abstract. The paper deals with the key features of rock behavior and their manifestations in loading diagrams. Mathematical models are proposed for describing elastic-plastic deformation, creep, and fracture of rocks. Deformation beyond the elastic limit is described using a model based on the combined yield surface and the nonassociated flow rule. The yield surface consists of tension cut-off segment, modified linear segments of the Drucker–Prager criterion and cup. The dilatancy coefficient depends on pressure and volumetric deformation. Model equations are derived to describe deformation in the dilation and compaction modes, as well as strain localization and fracture with consideration for damage kinetics. Several examples of numerical modeling are given to illustrate these phenomena.

Citation: Rev. Adv. Mater. Technol., 2023, vol. 5, no. 3, pp. 30–38

View online: https://doi.org/10.17586/2687-0568-2023-5-3-30-38

View Table of Contents: https://reviewsamt.com/issues

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